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Diffractive b-Space Peripherality and Nuclear Coherent Production

URI MAOR*
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and
Department of Physics and Astronomy
Tel-Aviv University, Tel-Aviv

ABSTRACT

The implication of b-space diffractive amplitude peripherality on nuclear coherent production is examined. It is argued that recent experimental estimates of diffractive multi-hadron total cross sections on nucleons should be upgraded.

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Experiments on coherent production of diffractive unstable systems on nuclei[1,2] provide us with the only attainable procedure to estimate the total cross sections of such systems on nucleons. The technique is relatively simple. The propogation of diffractive states through nuclear matter is optically parametrized,[3] and one extracts the desired cross sections by fitting the A dependence of the measured coherent production off nuclei. The analysis[4] of a recent experiment on $\pi^+ A \to (3\pi) A$ went one step further. The (3π) system was decomposed to its partial waves using the Ascoli program[5] and the dominant 1^+ and smaller 0^- partial waves were extracted for each nuclei so as to obtain their total cross sections from the fitted A dependence. The results obtained are

$$\sigma_{tot}(3\pi, N) = 20.^{+1.8}_{-1.5} \text{ mb} \tag{1}$$

$$\sigma_{tot}(1^+, N) = 15.8^{+1.5}_{-1.3} \text{ mb} \tag{2}$$

$$\sigma_{tot}(0^-, N) = 49.^{+9.}_{-7.} \text{ mb} \tag{3}$$

Somewhat surprisingly one finds that $\sigma_{tot}(1^+, N)$ is quite smaller and $\sigma(0^-, N)$ very considerably bigger than $\sigma_{tot}(\pi^+, N)$.

One may question the ability of the Ascoli program (which depends on many assumptions) to properly project the small 0^- wave. However, estimates (1) and (2), which relate to the bulk of the experimental data, yield cross sections which are quite smaller than the $\pi^+ N$ total cross section. In the following we shall attempt to assess the validity of these estimates by re-examining the theoretical input of the optical model parametrization of coherent diffractive production.[3] This parametrization is the key to the experimental analysis and in our opinion one of its main ingredients should be changed. Such a change results in an upgrade trend of the estimated total cross sections of unstable hadron systems on nucleons. Our comments relate, therefore, also to the estimates[2] of total baryonic diffractive systems cross section on nucleons.

Let us briefly review the extension of the Glauber theory[6] to account for coherent diffractive production off a nucleus A.[3] The production amplitude for small momentum transfers is given in the high energy limit by:

$$F_c(\vec{q}) = \frac{ik}{2\pi} \sum_{j=i}^{A} \int e^{i\vec{q}\cdot\vec{b}}\, d^2b \prod_{\ell=1}^{A} \rho(\vec{r}_\ell)\; \Gamma_{12}(\vec{b}-\vec{s}_j)$$
$$\times \prod_{z_i<z_j} \left[1 - \Gamma_{11}(\vec{b}-\vec{s}_i)\right] \prod_{z_k>z_j} \left[1 - \Gamma_{22}(\vec{b}-\vec{s}_k)\right]$$
$$\times\, d\vec{r}_1 \ldots d\vec{r}_A \quad , \tag{4}$$

where $\vec{q} = \vec{p}_1 - \vec{p}_2$ is the momentum transfer. k is the common magnitude of $\vec{p}_1$ and $\vec{p}_2$. $\vec{b}$ is the impact vector of particle 1 (incoming) or 2 (outgoing). $\vec{r}_i = (\vec{b}_i, z_i)$ is the i-th target nucleon position vector.

$\Gamma_{xy}(\vec{b})$ and $f_{xy}(\vec{q})$ are the two-body scattering amplitudes in b and q space respectively. They are related by the standard transformation

$$f_{xy}(\vec{q}) = \frac{ik}{2\pi} \int e^{i\vec{q}\cdot\vec{b}} \, \Gamma_{xy}(\vec{b}) \, d^2b \quad . \tag{5}$$

$\rho(\vec{r}_i) = \rho(\vec{b}_i, z_i)$ is the (normalized) single particle density function of the nucleus A.

Approximating the scattering amplitude (4), Kölbig and Margolis[3] evaluate integrals of the form:

$$\int \Gamma_{xy}(\vec{b}-\vec{s}) \, \rho(\vec{s}, z) \, d^2s \, dz \approx$$

$$\approx \int \Gamma_{xy}(\vec{b}-\vec{s}) \, d^2s \int_{-\infty}^{\infty} \rho(\vec{b}, z) \, dz = -\frac{2\pi i}{k} \, f_{xy}(0) \, \frac{T(\vec{b})}{A} \, , \tag{6}$$

where:

$$T(\vec{b}) = A \int_{-\infty}^{\infty} \rho(\vec{b}, z) \, dz \, . \tag{7}$$

We note that in the high energy limit $\rho(\vec{r}) = \rho(r)$ and $T(\vec{b}) = T(b)$.

The explicit approximation made in (6) is that $\rho(\vec{s}, z)$ is slowly varying compared with $\Gamma_{xy}(\vec{b}-\vec{s})$ as a function of $\vec{s}$. <u>Implicitly one assumes also that $\Gamma_{xy}(\vec{b}-\vec{s})$ is a central profile peaking at $b = 0$.</u> This central b-space behaviour has been established for the known elastic scattering profiles $\Gamma_{11}(b)$ and probably can be assumed also for $\Gamma_{22}(b)$ on which we have no direct information. This is, however, not an appropriate assumption for $\Gamma_{12}(b)$, the profile of the two body diffractive amplitude, which is peripheral in b-space. This is particularly so for $\pi N \rightarrow (3\pi)N$ which is highly peripheral due to its t-channel helicity conservation property.

If $\Gamma_{12}(\vec{b})$ actually peaks at $b = b_0$ and if $\rho(\vec{b}, s)$ is indeed slowly varying compared to $\Gamma_{12}(\vec{b} - \vec{s})$ as a function of $\vec{s}$, we expect

$$\int \Gamma_{12}(\vec{b} - \vec{s})\, \rho(\vec{s}, z)\, d^2 s\, dz \approx -\frac{2\pi i}{k}\, f_{12}(0)\, \frac{T(b - b_0)}{A} \,. \tag{8}$$

In the high energy limit, with purely imaginary amplitudes, we get-following the standard procedures[3]

$$\frac{d\sigma_c}{dt} = \left(\frac{d\sigma_0}{dt}\right)_{t=0} \left|\tilde{F}_c\right|^2 , \tag{9}$$

where

$$\tilde{F}_c = \frac{2}{\sigma_1 - \sigma_2} \int J_0(qb) \left[e^{-\frac{1}{2}\sigma_1 T(b)} - e^{-\frac{1}{2}\sigma_2 T(b)} \right] \frac{T(b - b_0)}{T(b)}\, d^2 b \,. \tag{10}$$

σ_0 is the diffractive cross section for $1N \rightarrow 2N$ and σ_i is the total cross section for $iN \rightarrow iN$. Our expression for $\tilde{F}_c$ is identical to the standard optical model result,[3] with the exception that the integrand of (10) is modulated by $T(b - b_0)/T(b)$ to account for the different b-space properties of Γ_{12} and Γ_{ii}. The actual experimental fitting is done with a finite energy corrected formula[3] for which we suggest the same modulation with the multiplicative factor $\rho(b - b_0, z)/\rho(b, z)$.

In order to exemplify the numerical significance of our suggested modification, we have calculated the effective nucleon number N defined by

$$\left(\frac{d\sigma_c}{dt}\right)_{t=0} = \left(\frac{d\sigma_0}{dt}\right)_0 \left| N\left(A, \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2, b_0\right)\right|^2 , \tag{11}$$

where:

$$N\left(A, \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2, b_0\right) = \frac{2}{\sigma_2 - \sigma_1} \int \left[e^{-\frac{1}{2}\sigma_1 T(b)} - e^{-\frac{1}{2}\sigma_2 T(b)} \right] \times$$

$$\times \frac{T(b - b_0)}{T(b)} \, d^2 b \, . \tag{12}$$

For $b_0 = 0$, Eq. (12) is reduced to the old definition[3] of N.

For simplicity we have considered in our calculations a gaussian nuclear density

$$T(b) = \frac{A}{\pi R^2} \exp\left(-\frac{b^2}{R^2}\right) , \tag{13}$$

with $R = 1.05\,A^{1/3}$. Such a simple parametrization was shown[7] to be sufficient for practical uses. Should one aim at a "best fit" it is desired of course to use[1,2] the finite energy expression with the slightly more accurate Fermi distribution for $\rho(r)$.

Some of our results, relevant to the reaction $\pi A \rightarrow (3\pi)A$, are summarized in Table I. We have taken $\sigma_1 = 25.4$ mb and then calculated N^2 from Eq. (12) for $\sigma_2 = 16$, 20 and 24 mb with Γ_{12} peaking at three values of $b_0 = 0$, 0.8 and 1.0 fermi. As can be seen the effects of $b_0 \neq 0$ are not negligible, in particular for the light and medium nuclei. Once $A > 100$ the effect becomes rather small. Present estimates of $\sigma(3\pi, N)$ have been carried out[1,4] on nuclei with $10 \leq A \leq 200$ with most of the data points coming from nuclei with $A < 100$. We conclude therefore that present estimates of σ_2 should be upgraded. Actually with our rather simplified parametrization we find that $\sigma(1^+, N)$ should be upgraded from 15.8 mb[4] to about 22 mb which is a big change. We further demonstrate our point of view in Figs. 1 and 2 where the A dependence of the coherent reactions $\pi A \rightarrow (3\pi)A$ and $nA \rightarrow (p\pi^-)A$ are displayed. Once again, it can be seen that an upgrade of 20-30% in the estimates of σ_2 may be required.

The present note did not aim at actually establishing new values for σ_2. Our calculations demonstrate, nevertheless, the sensitivity of σ_2 fits to the fine details of the input assumptions. As we have seen, proper evaluation of integrals like Eq. (6) is crucial for a reliable coherent production cross section calculation. For this purpose the δ-function approximation originally used[3] is rather crude for either central or peripheral $\Gamma_{xy}(b)$ distributions. This is particularly so when the calculation applies to nuclei with $A < 100$. We have recalculated Eq. (6) explicitly with a variety of gaussian Γ_{xy} distributions and then used the results as input for the calculation of Eqs. (10) and (12). We conclude that an exact evaluation of (6) is rather important for light and medium nuclei but it does not change our qualitative observation on the importance of considerating $\Gamma_{12}(b)$ peripherality. The practical problem is that although $\Gamma_{11}(b)$ is relatively well known, we have only partial knowledge on $\Gamma_{12}(b)$ and none on $\Gamma_{22}(b)$. Sensible input would be to assume that $\Gamma_{22}(b)$ is proportional to $\Gamma_{11}(b)$ for which a central gaissian distribution is quite proper. $\Gamma_{12}(b)$ is known for $\pi N \rightarrow (3\pi)N$ and then the calculation of (10) or (12) is straightforward. For other reactions, when $\Gamma_{12}(b)$ is not known, any σ_2 estimate is model dependent, though the general assumption of b-space Γ_{12} peripherality is probably correct.

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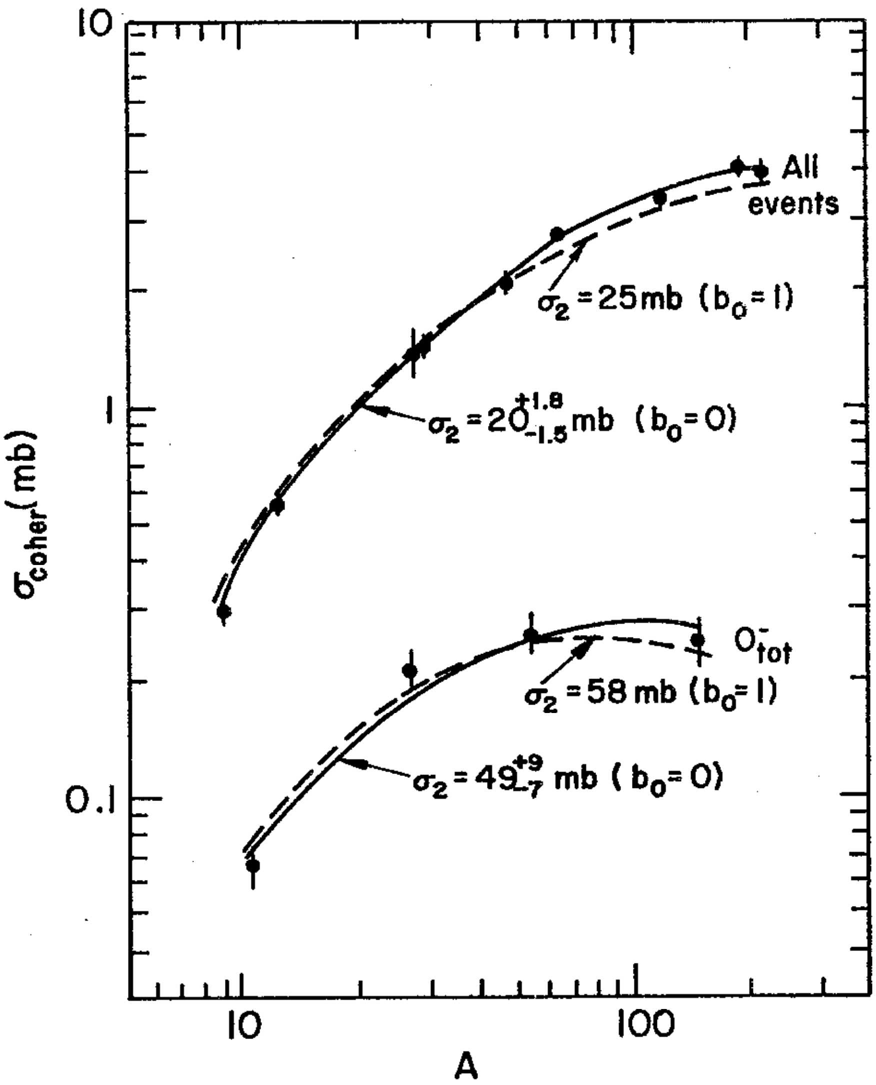
A	$\sigma_2 = 16$ mb			$\sigma_2 = 20$ mb			$\sigma_2 = 24$ mb		
	$b_0 = 0.$	$b_0 = .8$	$b_0 = 1.$	$b_0 = 0.$	$b_0 = .8$	$b_0 = 1.$	$b_0 = 0.$	$b_0 = .8$	$b_0 = 1.$
10	41.2	45.0	47.1	38.0	41.9	44.1	35.2	39.1	41.3
27	220.0	233.9	241.5	197.9	211.8	219.5	179.0	192.8	200.4
40	409.2	429.2	440.1	363.7	383.5	394.5	325.3	345.0	355.8
64	797.9	821.8	634.9	695.7	719.5	732.6	611.8	635.3	648.2
140	1707.9	1713.2	1716.6	1405.8	1413.2	1417.8	1173.0	1181.6	1186.7

Table I: $N^2\left(A, \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2, b_0\right)$ values are tabulated for various nuclei with $\sigma_1 = 25.4$ mb and $\sigma_2 = 16, 20, 24$ mb and $b_0 = 0, .8, 1.$ fermi.

FIGURE CAPTIONS

Fig. 1: Production cross sections of the 3π system and 0^- state for different target nuclei at 15.1 GeV/c. Full lines correspond to the standard calculation ($b_0 = 0$). Dotted lines correspond to the modified calculation with $b_0 = 1$ fermi. $\sigma_1 = 25.4$ mb in both sets of calculations. Data from Ref. 4.

Fig. 2: Production cross sections of the $p\pi^-$ system for different target nuclei at 6-16 GeV/c. Full line corresponds to the standard calculation ($b_0 = 0$). Dotted line corresponds to the modified calculation with $b_0 = 1$ fermi. $\sigma_1 = 39$ mb in both calculations. Data from Ref. 2.



10
1
0.1
σ_{coher} (mb)
All events
$\sigma_2 = 25$ mb ($b_0 = 1$)
$\sigma_2 = 20^{+1.8}_{-1.5}$ mb ($b_0 = 0$)
O^-_{tot}
$\sigma_2 = 58$ mb ($b_0 = 1$)
$\sigma_2 = 49^{+9}_{-7}$ mb ($b_0 = 0$)
10
100
A

FIGURE 1

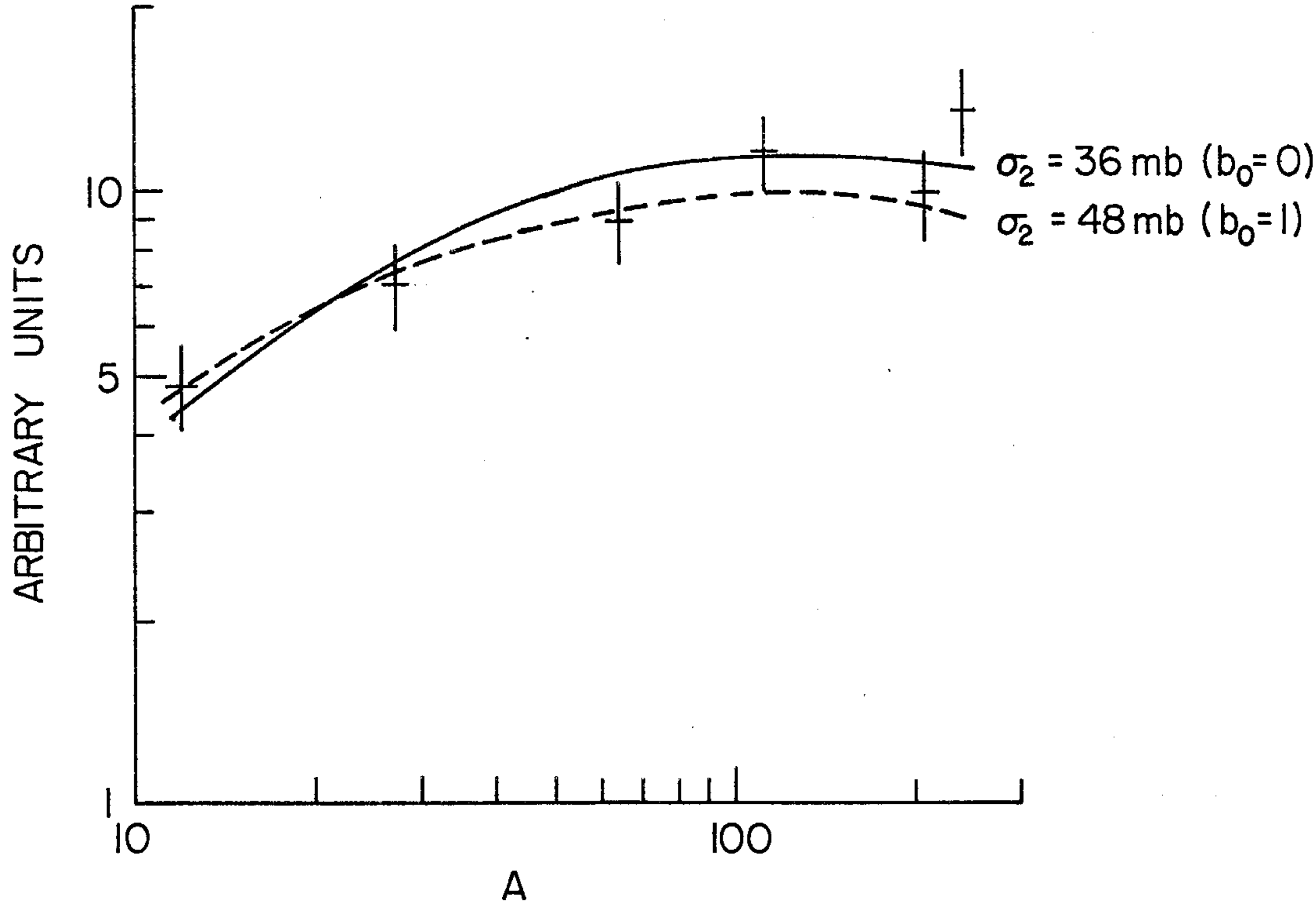

ARBITRARY UNITS
10
5
1
10
100
A
σ_2 = 36 mb (b_0= 0)
σ_2 = 48 mb (b_0= 1)

FIGURE 2